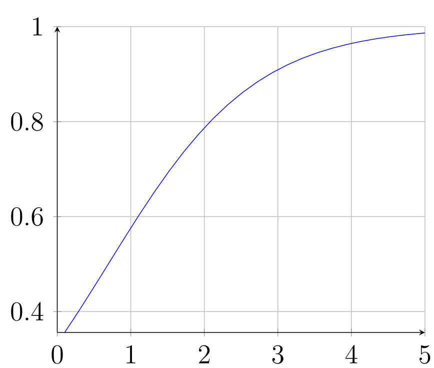
Math 208 I, Midterm 1 Name: ____________________

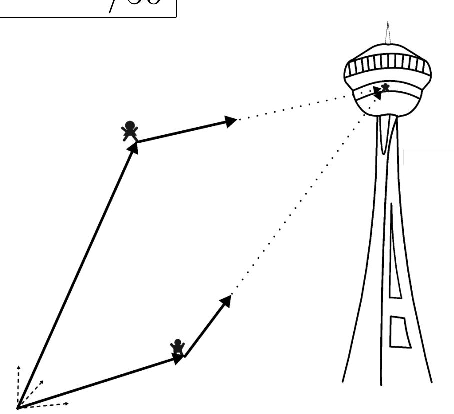
Signature: ____

Student ID #: ______________ Section #: ______________

- You are allowed a Ti-30x IIS Calculator and one 8.5×11 inch paper with handwritten notes on both sides. Other calculators, electronic devices (e.g. cell phones, laptops, etc.), notes, and books are **not** allowed.
- Some questions require you to explain answers: no explanation, no credit.
- Try to show your work on all questions: no work, no partial credit.
- You may use the back of the exam for scratch work: please submit any additional paper you use.
- Place a box around your answer to each question.
- Raise your hand if you have a question.

1	/10
2	/10
3	/10
4	/10
5	/10
T	/50





Good Luck!

(1) A logistic curve is a curve in the (x, y)-plane defined by an equation of the form $y(1 + ae^{-x}) = b$. (See coverpage for an illustration.)

(a) (4pts) Write a system of linear equations in a, b that can be used to fit a logistic curve to the following values of (x, y): $(0, 1/3), (\ln 2, 1/2)$.
(No need to simplify...yet.)

(b) (4pts) Solve this system (Hint: recall $e^0 = 1$, and $e^{-\ln x} = 1/x$.)

(c) (2pts) How many equations can we add to this system without violating the existence of a solution? Explain.

(2) (a) (7pts) Determine a 2×3 matrix $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ in *reduced echelon form*, such that z is a free variable and such that

$$\mathbf{x} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

is the general solution to the system

$$\begin{aligned} ax + by + cz &= -1 \\ dx + ey + fz &= 2. \end{aligned}$$

(b) (3pts) Consider the linear transformation associated to this matrix:

$$T_A(x, y, z) = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \end{pmatrix}$$

Calculate $T_A(1, 1, 1)$.

(3) In each case below describe all values of t (when possible) for which the given vectors are linearly **dependent**. (2.5 pts each)

(a) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} \pi \\ 4 \end{pmatrix}, \begin{pmatrix} \sqrt{2} \\ 2024 \end{pmatrix}, \begin{pmatrix} t \\ 7 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ t \\ 5 \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1+t \end{pmatrix}, \begin{pmatrix} 1 \\ t^2-3 \\ \cos(t) \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} t \\ t \\ 2 \\ 0 \end{pmatrix}$

(4) Three friends go to the space needle. In geocentric coordinates, Emmy and Johann stand at positions $\mathbf{x}_1 = (2, 0, 0)$, $\mathbf{x}_2 = (1, 1, 0)$, respectively, and stare in the direction of vectors $\mathbf{v}_1 = (1, 2, 3)$, $\mathbf{v}_2 = (1, 1, 2)$, respectively, towards Olga on the observation deck (see coverpage for an illustration.)

(a) (4pts) Model this problem with a system of 3 equations in 2 unknowns.

(b) (4pts) Calculate Olga's position vector $\mathbf{x}_3$.

(c) (2pts) Let A be the 3×2 coefficient matrix of the system from part a, and consider the associated linear transformation. Is T_A 1-1? Explain.

(5) (a) (4pts) Write down the matrix representation of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that sends a point $(x, y) \in \mathbb{R}^2$ to the closest point on the x-axis.

(b) (2pts) Is the linear transformation from 5a) onto? Explain.

(c) (4pts) Write down the matrix representation of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects a vector across the y-axis, then rotates it 270° counterclockwise around the origin.